

Fingerprints of classical diffusion in open 2D mesoscopic systems in the metallic regime

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Abstract. – We investigate the distribution of the resonance widths $\mathcal{P}(\Gamma)$ and Wigner delay times $\mathcal{P}(\tau_W)$ for scattering from two-dimensional systems in the diffusive regime. We obtain the forms of these distributions (log-normal for large τ_W and small Γ , and power law in the opposite case) for different symmetry classes and show that they are determined by the underlying diffusive classical dynamics. Our theoretical arguments are supported by extensive numerical calculations.

Quantum scattering has been a subject of intensive research activity both in mesoscopic physics and in Quantum Chaos in the last years [1–7]. Among the most interesting quantities for the description of a scattering process are the Wigner delay times and resonance widths. The former quantity captures the time-dependent aspects of quantum scattering. It can be interpreted as the typical time an almost monochromatic wave packet remains in the interaction region. It is related to the energy derivative of the total phase shift $\Phi(E) = -i \ln \det S(E)$ of the scattering matrix $S(E)$, i.e. $\tau_W(E) = \frac{d\Phi(E)}{dE}$. Resonances are defined as poles of the S -matrix occurring at complex energies $\mathcal{E}_n = E_n - \frac{i}{2}\Gamma_n$, where E_n is the position and Γ_n the width of the resonance. They correspond to "eigenstates" of the open system that decay in time due to the coupling to the "outside world".

For chaotic/ballistic systems Random Matrix Theory (RMT) is applicable, and the distributions of resonance widths $\mathcal{P}(\Gamma)$ and Wigner delay times $\mathcal{P}(\tau_W)$ are known [2]. As the disorder increases the system becomes diffusive and the deviations from RMT become increasingly apparent. In the strongly disordered limit where localization dominates, the distribution of resonances $\mathcal{P}(\Gamma)$ [4] and delay times $\mathcal{P}(\tau_W)$ [5] were found recently. At the same time, an attempt to understand systems at critical conditions, was undertaken in [6, 7]. For diffusive mesoscopic samples, however, there is no study of $\mathcal{P}(\Gamma)$ and $\mathcal{P}(\tau_W)$ besides Ref. [3] where the authors have focused on the tails of $\mathcal{P}(\Gamma)$ for a quasi-1D system in the diffusive regime. This study is important for diffusive random lasers, where the knowledge of short resonance width distribution determines the properties of lasing thresholds [8], as well as for various other applications like mesoscopic capacitors [9], microwave cavities [10] and chaotic optical cavities [11] where most of the theoretical treatment is limited by RMT. We point here that

current developments of microwave experiments in random dielectric media in the diffusive regime [12] may allow us a direct comparison between theory and experiment. Finally we note that an analogous theoretical study for the distribution of wavefunction intensities of *closed* systems was put forward very early in the development of the mesoscopic physics [13] and recently their statistical properties have become clear [14–18].

In this paper, we study $\mathcal{P}(\Gamma)$ and $\mathcal{P}(\tau_W)$ for $2D$ open systems in the diffusive regime. We will show that these distributions are determined by the diffusive classical dynamics of the corresponding closed system and depend on the time-reversal symmetry (TRS). Specifically, the resonance width distribution $\mathcal{P}(\Gamma)$ is given by

$$\begin{aligned} \mathcal{P}(\Gamma < \Gamma_{cl}) &\sim \exp(-C_\beta (\ln \Gamma)^2), \quad \text{where } C_\beta \sim \beta D \\ \mathcal{P}(\Gamma \gtrsim \Gamma_{cl}) &\sim \sqrt{\frac{D}{L^2}} \frac{1}{\Gamma^{3/2}} \end{aligned} \quad (1)$$

where D is the classical diffusion coefficient (which is proportional to the Thouless conductance for disordered $2D$ systems), L is the linear size of the system, Γ_{cl} is the classical decay rate [19] which is inversely proportional to the Thouless time L^2/D and β denotes the symmetry class [$\beta = 1(2)$ for preserved (broken) TRS]. For the distribution of Wigner delay times we obtain

$$\begin{aligned} \mathcal{P}(\tau_W \lesssim \Gamma_{cl}^{-1}) &\sim \frac{1}{\tau_W^{3/2}} \exp(-\sigma/\tau_W) \\ \mathcal{P}(\tau_W > \Gamma_{cl}^{-1}) &\sim \exp(-C_\beta (\ln \tau_W)^2) \end{aligned} \quad (2)$$

where σ is some constant of order unity. Our results (1,2) are applicable both for chaotic and disordered systems, provided that: (i) transport between scattering events may be treated semiclassically, (ii) a particle scatters many times while it traverses the system and (iii) strong localization effects can be neglected.

To numerically demonstrate the above predictions we use a model of the $2D$ Kicked Rotor (KR) with absorbing boundary conditions. The corresponding closed system is described by the time-dependent Hamiltonian [17,20]

$$H = H_0 + kV \sum_m \delta(t - m), \quad H_0(\{\mathcal{L}_i\}) = \sum_{i=1}^2 \frac{\rho_i}{2} \mathcal{L}_i^2 \quad (3)$$

$$V(\{\theta_i\}) = \cos(\theta_1) \cos(\theta_2) \cos(\alpha) + \frac{1}{2} \sin(2\theta_1) \cos(2\theta_2) \sin(\alpha)$$

where \mathcal{L}_i denotes the angular momentum and θ_i the conjugate angle of one rotor. Here the kick period is one, k is the kicking strength, while ρ_i is a constant inversely proportional to the moment of inertia of the rotor. The parameter α breaks TRS. The motion generated by (3) is classically chaotic and for a sufficiently strong kicking strength k there is diffusion in momentum space with diffusion coefficient $D \equiv \lim_{t \rightarrow \infty} \langle \mathcal{L}^2(t) \rangle / t \simeq \frac{k^2}{2}$ [20].

Quantum mechanically, this system can be described by a finite-dimensional evolution operator for one period

$$U = e^{(-iH_0(\{\mathcal{L}_i\})/2)} e^{(-iV(\{\theta_i\}))} e^{(-iH_0(\{\mathcal{L}_i\})/2)} \quad (4)$$

where we put $\hbar = 1$. The eigenvalues of U are $\lambda_n = e^{i\omega_n}$ and lie on the unit circle; ω_n are known as quasi-energies and are dimensionless. The corresponding mean quasi-energy spacing is $\Delta = 2\pi/L^2$, where L is the linear size of the system. A detailed presentation of our model

can be found in [17], where the statistical properties of wavefunction intensities in the regime of interest were investigated and found to be in agreement with the theoretical predictions for mesoscopic diffusive samples [14, 15]. Therefore the 2D KR is a representative model both for 2D disordered and chaotic systems.

In this paper, we turn the closed 2D KR model into an open one. To this end we impose absorption at the boundary of a square sample of size $L \times L$ in the momentum space. In other words, every time that one of the components of the two dimensional momentum $(\mathcal{L}_1, \mathcal{L}_2)$ takes on the value 1 or L , the particle is absorbed without coming back to the sample. Using a recently proposed recipe [21] we can write down the corresponding scattering matrix S in the form [22]

$$S(\omega) = -WUe^{i\omega} \frac{1}{I - e^{i\omega}PU} W^\dagger, P = I - W^\dagger W \quad (5)$$

where I is the $L^2 \times L^2$ unit matrix and W is a $M \times L^2$ matrix. It has only M non-zero elements which are equal to one and describe at which "site" of the $L \times L$ sample we attach M "leads" [in our case $M = 4(L - 1)$]. Here $W^\dagger W$ is a projection operator onto the boundary, while P is the complementary projection operator. The scattering matrix S_{ij} given by Eq. (5) can be interpreted in the following way: once a wave enters the sample, it undergoes multiple scattering induced by $[I - e^{i\omega}PU]^{-1} = \sum_{n=0}^{\infty} (e^{i\omega}PU)^n$ until it is transmitted out. It is clear therefore that the matrix $\tilde{U} = PU$ propagates the wave inside the sample. However, contrary to the closed system in which the evolution operator is unitary, the absorption breaks the unitarity of the evolution matrix \tilde{U} so that all eigenvalues $\tilde{\lambda}$ move inside the unit circle. Therefore each eigenvalue can be written in the form $\tilde{\lambda}_n = e^{i\tilde{\omega}_n} = \exp(-i\omega_n - \Gamma_n/2)$ where $\Gamma_n > 0$ is the dimensionless resonance width of an eigenstate.

The Wigner delay time can be expressed as the sum of proper delay times τ_q . The latter are the eigenvalues of the Wigner-Smith operator written in our case as [22]

$$Q(\omega) \equiv \frac{1}{i} S^\dagger \frac{dS}{d\omega} = -e^{-i\omega} W K^\dagger W^\dagger W K U^\dagger K W^\dagger \quad (6)$$

where $K = (P - U^\dagger e^{-i\omega})^{-1}$.

Below we present our theoretical considerations and compare them with the numerical data obtained for the 2D KR model. The parameters of the model were chosen in such a way that the conditions (i)-(iii) discussed above were fulfilled (see [17]). In order to improve our statistics, we randomized the phases of the kinetic term of the evolution operator (4) and used a number of different realizations. In all cases we had at least 60000 data for statistical processing.

Resonance widths distribution. We start our analysis with the study of resonance width distribution $\mathcal{P}(\Gamma)$ for $\Gamma < \Gamma_{cl}$. The small resonances $\Gamma < \Delta$ can be associated, with the existence of pre-localized states of the "isolated" system. The latter states show strong localization features in contrast to the typical states in the diffusive regime and are considered as precursors of localization. They consist of a short-scale bump (where most of the norm is concentrated) and they decay rapidly in a power law from their center of localization [14, 15]. One then expects that states of this type with localization centers at the bulk of the sample are affected very weakly by the opening of the system at the boundaries. In first order perturbation theory, considering the opening as a small perturbation we obtain

$$\frac{\Gamma}{2} = \langle \Psi | W^\dagger W | \Psi \rangle = \sum_{n \in \text{boundary}} |\Psi(n)|^2 \sim L |\Psi(L)|^2 \quad (7)$$

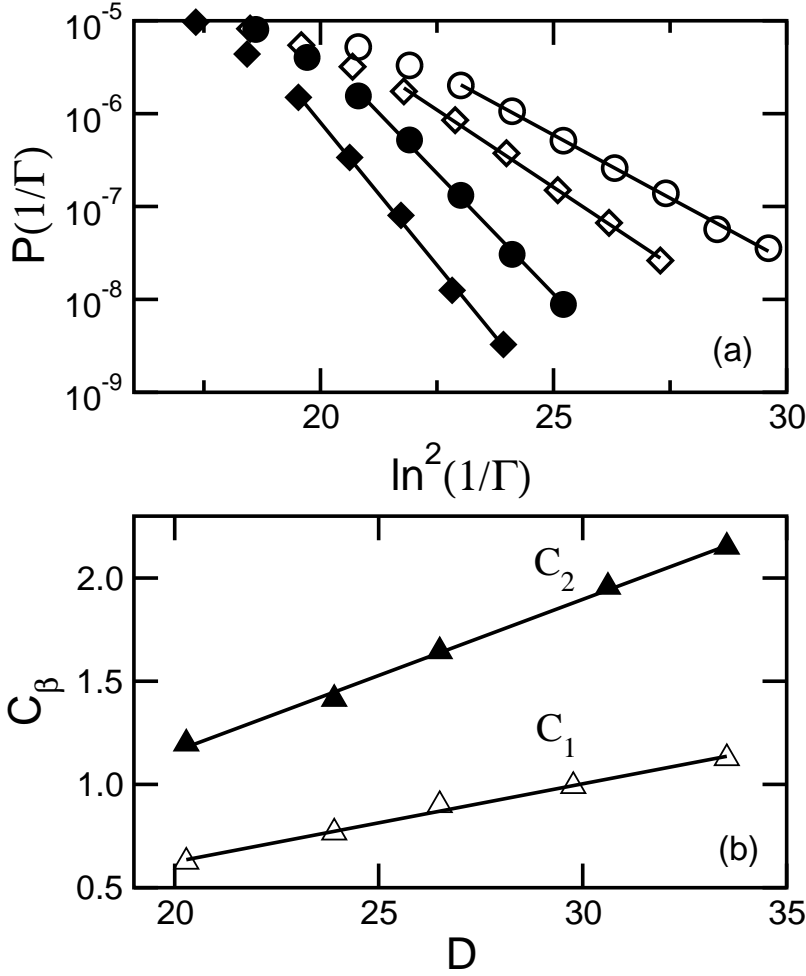


FIG.1: (a) The distribution of resonance widths (plotted as $\mathcal{P}(1/\Gamma)$ vs. $1/\Gamma$) for $\Gamma < \Gamma_{cl}$ for two representative values of D . The system size in all cases is $L = 80$. Filled symbols correspond to broken TRS. The solid lines are the best fit of Eq. (1) for $\beta = 1(2)$ to the numerical data. (b) Coefficients C_β vs. D . The solid lines are the best fits to $C_\beta = A_\beta D + B_\beta$ for $\beta = 1(2)$. The ratio $R = A_2/A_1 = 1.95 \pm 0.03$

where $|\Psi(L)|^2$ is the wavefunction intensity of a pre-localized state at the boundary. At the same time the distribution of $\theta = 1/\sqrt{L}\Psi(L)$ for large values of the argument is found to be of log-normal type i.e. $\mathcal{P}(\theta) \sim \exp(-\pi^2 D \ln^2(\theta^2))$ [15]. Using this together with Eq. (7) we get $\mathcal{P}(1/\Gamma) \sim \exp(-\pi^2 D \ln^2(1/\Gamma))$. We would like to stress that the expression for $\mathcal{P}(\theta)$, must be corrected by including the TRS factor β in the exponent. This is due to the fact that the Optimal Fluctuation Method, which was used to derive the above expression for $\mathcal{P}(\theta)$, does not describe the effect of breaking TRS in a correct way [17,18]. Taking all the above into account we end up with the expression given in Eq. (1).

The numerical data reported in Fig. 1 support the validity of the above considerations. However, we mention that the perturbative argument is valid only for the case of very small resonances i.e. $\Gamma < \Delta$, whereas our numerical data indicate that one can extend the log-normal behaviour of $\mathcal{P}(\Gamma)$ up to resonances with $\Delta < \Gamma < \Gamma_{cl}$.

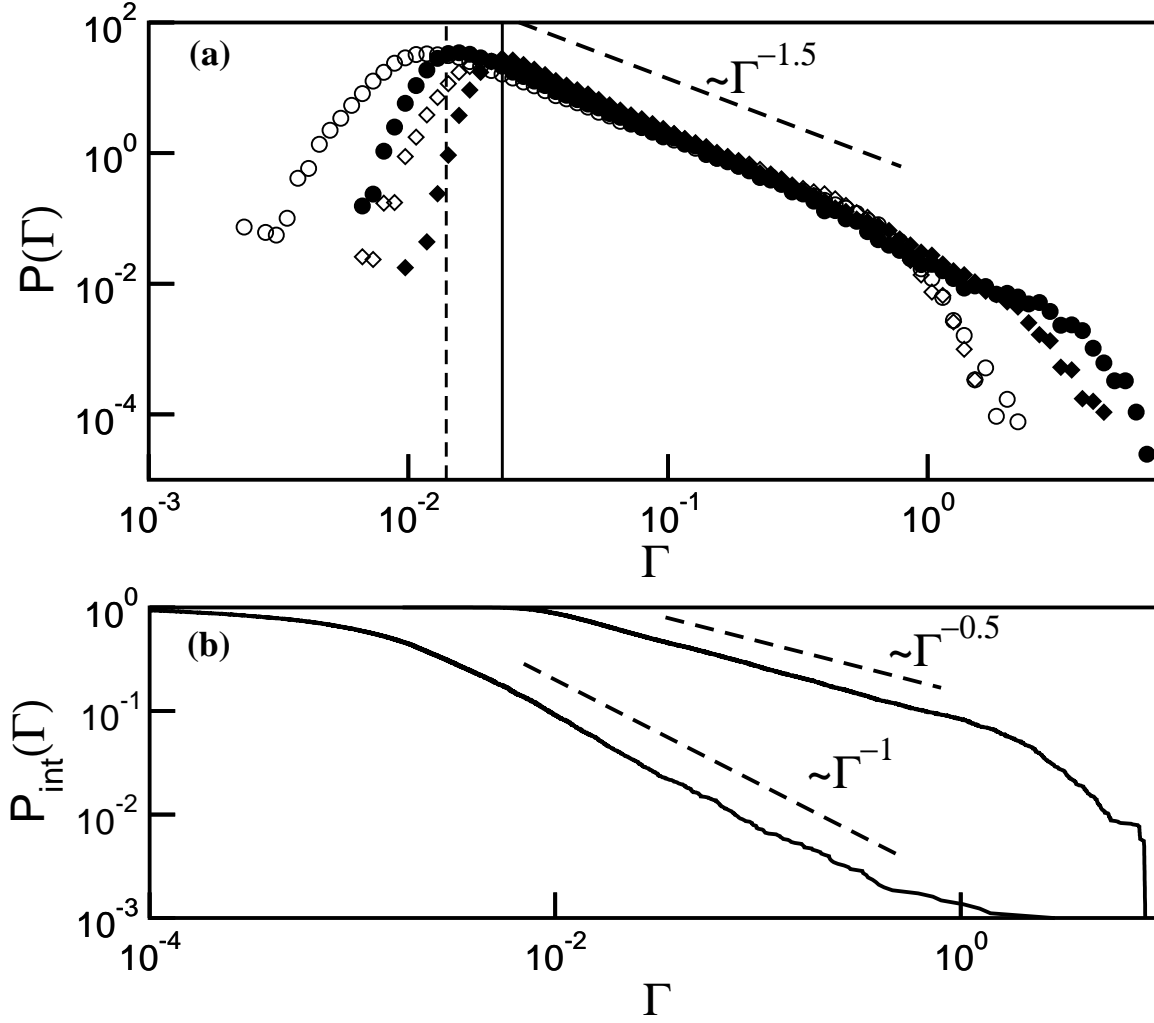


FIG. 2: (a) Resonance width distribution $\mathcal{P}(\Gamma)$ for preserved TRS and $D = 20.3$ (\circ) and $D = 33.5$ (\diamond). The corresponding filled symbols represent $\mathcal{P}(\Gamma)$ for broken TRS and the same values of D . The dashed (solid) vertical line mark the classical decay rate Γ_{cl} [19] for $D = 20.3(33.5)$. (b) The $\mathcal{P}_{\text{int}}(\Gamma)$ for a sample with nine leads (lower curve). For comparison we plot also the $\mathcal{P}_{\text{int}}(\Gamma)$ for the same sample but when we open the system from the boundaries. The dashed lines correspond to the theoretical predictions (9) and (10).

Next we turn to the analysis of $\mathcal{P}(\Gamma)$ for $\Gamma \gtrsim \Gamma_{cl}$. In this case, the resonances are overlapping, the corresponding eigenstates are strongly non-orthogonal and first-order perturbation theory breaks down [23]. In Fig. 2a we report our numerical results for $\mathcal{P}(\Gamma)$ with preserved (broken) TRS for two representative values of D . An inverse power law $\mathcal{P}(\Gamma) \sim \Gamma^{-1.5}$ is evident in accordance with Eq. (1) [24]. From Fig. 2a it is clear that this part of the distribution is independent of the symmetry class, in contrast to the small resonance distribution.

The following argument provides some understanding of the behaviour of $\mathcal{P}(\Gamma)$ for $\Gamma \gtrsim \Gamma_{cl}$. First we need to recall that the inverse of Γ represents the quantum lifetime of a particle in the corresponding resonant state escaping into the leads. Moreover we assume that the particles are uniformly distributed inside the sample and diffuse until they reach the boundaries, where

they are absorbed. Then we can associate the corresponding lifetimes with the time $t_R \sim 1/\Gamma_R \sim R^2/D$ a particle needs to reach the boundaries, when starting distance R away. This classical picture can be justified for all states with $\Gamma \gtrsim \Gamma_{cl} \sim D/L^2$. The relative number of states that require a time $t < t_R$ in order to reach the boundaries (or equivalently the number of states with $\Gamma > \Gamma_R$) is

$$\mathcal{P}_{\text{int}}(\Gamma_R) = \int_{\Gamma_R}^{\infty} \mathcal{P}(\Gamma) d\Gamma \sim \frac{S(t_R)}{L^2} \quad (8)$$

where $S(t_R)$ is the area populated by all particles with lifetimes $t < t_R$. In the case of open boundaries we get

$$\mathcal{P}_{\text{int}}(\Gamma_R) \sim \frac{L^2 - (L - 2R)^2}{L^2} \sim \sqrt{\frac{\Gamma_{cl}}{\Gamma_R}} - \frac{\Gamma_{cl}}{\Gamma_R}. \quad (9)$$

For $\Gamma_R > \Gamma_{cl}$ the first term in the above equation is the dominant one and thus Eq. (1) follows.

Here it is interesting to point that a different way of opening the system might lead to a different power law behaviour for $\mathcal{P}(\Gamma)$. Such a situation can be realized if instead of opening the system at the boundaries we introduce "one-site" absorber (or one "lead") somewhere in the sample. In such a case we have

$$\mathcal{P}_{\text{int}}(\Gamma_R) \sim \frac{S(t_R)}{L^2} = \frac{R^2}{L^2} = \frac{Dt_R}{L^2} \sim \frac{\Gamma_{cl}}{\Gamma_R}. \quad (10)$$

The above result is valid for any number M of "leads" such that the ratio M/L^2 scales as $1/L^2$. In Fig. 2b we report $\mathcal{P}_{\text{int}}(\Gamma)$ for the case with nine "leads" attached somewhere to the 2D sample.

A straightforward generalization of our arguments for 3D systems in the metallic regime gives $\mathcal{P}_{\text{int}}(\Gamma_R) \sim (\Gamma_{cl}/\Gamma_R)^{0.5} - 2(\Gamma_{cl}/\Gamma_R) + (4/3)(\Gamma_{cl}/\Gamma_R)^{1.5}$ which for $\Gamma_R > \Gamma_{cl}$ leads to the same universal expression as in Eq. (1). Similarly, the analogue of Eq. (10) in 3D is $\mathcal{P}_{\text{int}}(\Gamma_R) \sim (\Gamma_{cl}/\Gamma_R)^{1.5}$.

Wigner delay times distribution. Our theoretical understanding will be based on the following relation

$$\tau_W(\omega) = \sum_{n=1}^{L^2} \frac{\Gamma_n}{(\omega - \omega_n)^2 + \Gamma_n^2/4} \quad (11)$$

which connects the Wigner delay time and the poles of the S -matrix. Let us start with the far tails. It is evident that large times $\tau_W(\omega) \sim \Gamma_n^{-1}$ correspond to the cases when $\omega \simeq \omega_n$ and $\Gamma_n \ll 1$. Then for the distribution of delay times we obtain $\mathcal{P}(\tau_W) \sim \int d\Gamma \mathcal{P}(\Gamma) \Gamma \delta(\tau_W - 1/\Gamma)$. Using the small resonance width asymptotics given by Eq. (1) we get the log-normal law of Eq. (2).

Now we estimate the behaviour of $\mathcal{P}(\tau_W)$ for $\tau_W \lesssim \Gamma_{cl}^{-1}$. In this regime many short-living resonances contribute to the sum (11). We may therefore consider τ_W as a sum of many independent positive random variables each of the type $\tau_n = \Gamma_n x_n$, where $x_n = \delta\omega_n^{-2}$. Assuming further that $\delta\omega_n$ are uniformly distributed random numbers we find that the distribution $\mathcal{P}(x_n)$ has the asymptotic power law behaviour $1/x_n^{3/2}$. As a next step we find that the distribution $\mathcal{P}(\tau_n)$ decays asymptotically as $1/\tau_n^{3/2}$ where we use that $\mathcal{P}(\Gamma_n) \sim 1/\Gamma_n^{3/2}$. Then the corresponding $\mathcal{P}(\tau_W)$ is known to be a stable asymmetric Levy distribution $L_{\mu,1}(\tau_W)$ of index $\mu = 1/2$ [25] which has the form given in Eq. (2) at the origin. We point out here that the asymptotic behaviour $\mathcal{P}(\tau_W) \sim 1/\tau_W^{3/2}$ emerges also for chaotic/ ballistic systems where the assumption of uniformly distributed $\delta\omega_n$ is the only crucial ingredient (see for example [2]).

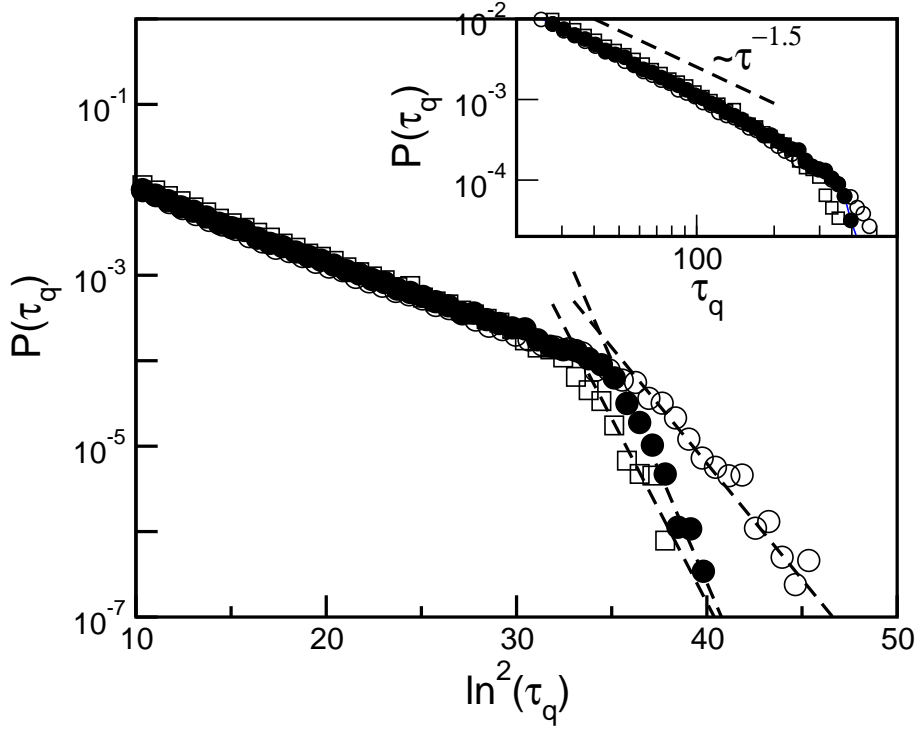


FIG. 3: The proper delay times distribution $\mathcal{P}(\tau_q)$ for $D = 20.3$ (\circ) and $D = 29.8$ (\square). The (\bullet) correspond to $D = 20.3$ but now with broken TRS. The dashed lines have slopes equal to C_β extracted from the corresponding $\mathcal{P}(\Gamma)$ (see Fig. 1b). In the inset we report $\mathcal{P}(\tau_q)$ for moderate values of τ_q in a double logarithmic scale.

Since $\tau_W = \sum_{i=1}^M \tau_q$, we expect the behaviour of the distribution of proper delay times $\mathcal{P}(\tau_q)$ to be similar to $\mathcal{P}(\tau_W)$ for large values of the arguments (for $\tau_W \gg 1$ we have $\tau_W \sim \tau_q^{\max}$). Moreover, from the numerical point of view $\mathcal{P}(\tau_q)$ can be studied in a better way because a larger set of data can be generated easily. Our numerical findings for $\mathcal{P}(\tau_q)$ are reported in Fig. 3 and are in nice agreement with Eq. (2), even for moderate values of τ_q . We stress here that the dashed lines in Fig. 3, have slopes equal to C_β taken from the corresponding log-normal tails of $\mathcal{P}(\Gamma)$.

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